MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2017/2018

EMT1026 – ENGINEERING MATHEMATICS II

(All Sections / Groups)

01 JUNE 2018

9.00 AM – 11.00 AM

(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **9 pages** (including cover page) with **4** Questions only.

2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.

3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

4. Several tables are provided in the **Appendix** for your reference.

Question 1

(a) Find the general solution of the following homogeneous differential equation:

$$3y''+10y'+3y=0$$ [3 marks]

(b) Hence, or otherwise, solve the following second order non-homogeneous differential equation:

$$3y''+10y'+3y=9x+5\cos x$$ [9 marks]

(c) Consider the following differential equation with variable coefficient:

$$y''-y'-4x^2y=0$$

Find the first six nonzero terms of the solution about $x_0=0$ by using power series method. [13 marks]

Question 2

(a) Consider the following heat conduction equation:

$$u_t=16u_{xx}$$

in the region $0\le x\le 16$, for $t>0$. The homogeneous boundary conditions are given by

$$u(0,t)=0 \text{ and}$$
$$u(16,t)=0.$$

Use the method of separation of variables to solve the given boundary value problem. [15 marks]

(b) Consider the following non-homogeneous initial boundary value problem:

$$16u_t=u_{xx}+16x,\quad 0\le x\le 16,\ \text{for } t>0$$
$$u(0,t)=0$$
$$u(16,t)=0$$
$$u(x,0)=0$$

Using the substitution $u(x,t)=w(x,t)+v(x)$, decompose the above problem into a homogeneous partial differential equation (PDE) and an ordinary differential equation (ODE). Then, solve the **ODE only**. [10 marks]

Continued...

Question 3

(**Note**: The tables in APPENDIX A and APPENDIX B may be useful for solving this question.)

(a) Find the Laplace transform of the following:

(i) $f(t) = e^{-5t}\sinh(2t)$ using the s-shifting property. [3 marks]

(ii) $g(t) = \frac{1}{2}t\cos t$ using the Derivative of Transform property. [4 marks]

(b) Find the inverse Laplace transform of the following:

(i) $F(s) = \frac{e^{-\pi s}}{s^2+1}$ using the t-shifting property. [3 marks]

(ii) $G(s) = \frac{10}{s(s^2+100)}$ using the Convolution Theorem. [4 marks]

(c) Using Laplace transform method, solve the following differential equation:

$y''(t) - 4y'(t) + 3y(t) = 8$, subject to $y(0) = 0, y'(0) = 0$. [11 marks]

Continued...

Question 4

(**Note**: The tables in APPENDIX C, APPENDIX D and APPENDIX E may be useful for solving this question.)

(a) Consider the probability density function for continuous random variable X,

$$f(x)=\begin{cases} k(x+1), & 0<x\le 2 \\ kx^2, & 2<x\le 3 \\ 0, & \text{otherwise.} \end{cases}$$

(i) Show that $k=\dfrac{3}{31}$. [4 marks]

(ii) Find $P(1<X\le 2.5)$. [3 marks]

(iii) Determine the mean of X. [3 marks]

(b) The average number of cars sold by the Pluto Company is 4 cars in every two days. Assume the sale is a Poisson distribution and one working week consists of 5 days. What is the probability that this company will sell

(i) less than 3 cars in 2 days? [4 marks]

(ii) a car in one working week? [3 marks]

(c) Suppose the number of games in which major league baseball players play during their careers is normally distributed with the mean equal to 1800 games and the standard deviation equal to 250 games.

(i) What is the probability that the players play in fewer than 780 games?

[4 marks]

(ii) What is the probability that the players play in more than 2200 games?

[4 marks]

Continued...

APPENDIX A:

Common Laplace Transform Pairs

	$f(t)$	$F(s)=\mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$
2.	t	$\frac{1}{s^2}$
3.	t^n	$\frac{n!}{s^{n+1}}$, $n=1,2,\ldots$
4.	e^{at}	$\frac{1}{s-a}$
5.	$\cos at$	$\frac{s}{s^2+a^2}$
6.	$\sin at$	$\frac{a}{s^2+a^2}$
7.	$\cosh at$	$\frac{s}{s^2-a^2}$
8.	$\sinh at$	$\frac{a}{s^2-a^2}$
9.	$u(t-a)$	$\frac{e^{-as}}{s}$
10.	$\delta(t-a)$	e^{-as}

Continued...

APPENDIX B:

Laplace Transform Properties

	Property Name	Formula
1.	Linearity	$L\{af(t)+bg(t)\}=aL\{f(t)\}+bL\{g(t)\}$
2.	s - shifting	$L\{e^{at}f(t)\}=F(s-a)$
3.	Transform of Derivative	$L\{f'(t)\}=sL\{f(t)\}-f(0)$ $L\{f''(t)\}=s^2L\{f(t)\}-sf(0)-f'(0)$
4.	Transform of Integration	$L\left\{\int_0^t f(\tau)d\tau\right\}=\frac{1}{s}F(s)$
5.	t - shifting	$L\{u(t-a)f(t-a)\}=e^{-as}F(s)$
6.	Differentiation of Transform	$L\{t.f(t)\}=-F'(s)$
7.	Integration of Transform	$L\left\{\frac{f(t)}{t}\right\}=\int_s^\infty F(\tilde{s})d\tilde{s}$
8.	Convolution Theorem	$L\{f(t)*g(t)\}=F(s).G(s)$, where $f(t)*g(t)=\int_0^t f(\tau)g(t-\tau)d\tau$.

Continued…

APPENDIX C:

Special Discrete Probability Distributions

Binomial Distribution, $X \sim b(x; n, p)$	
P.m.f.	$P(X = x) = {}^{n}C_{x} p^{x} q^{n-x}$, for $x = 0,1,\ldots,n$ and where $q = 1 - p$.
Mean	$E[X] = np$
Variance	$Var(X) = npq$
Hypergeometric Distribution, $X \sim h(x; N, n, k)$	
P.m.f.	$P(X = x) = \dfrac{{}^{k}C_{x} \times {}^{N-k}C_{n-x}}{{}^{N}C_{n}}$, for $x = 0,1,\ldots\min(n,k)$.
Mean	$E[X] = \dfrac{nk}{N}$
Variance	$Var(X) = \dfrac{nk}{N}\left(\dfrac{N-n}{N-1}\right)\left(1 - \dfrac{k}{N}\right)$
Poisson Distribution, $X \sim p(x; \lambda)$	
P.m.f.	$P(X = x) = \dfrac{\lambda^{x} e^{-\lambda}}{x!}$, for $x = 0,1,\ldots$.
Mean	$E[X] = \lambda$
Variance	$Var(X) = \lambda$

Continued...

APPENDIX D:

Special Continuous Probability Distributions

Continuous Uniform Distribution, $X \sim U(a,b)$	
P.d.f.	$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \dfrac{a+b}{2}$
Variance	$Var(X) = \dfrac{(b-a)^2}{12}$

Exponential Distribution, $X \sim \exp(1/\beta)$	
P.d.f.	$f_X(x) = \begin{cases} \dfrac{1}{\beta} e^{-x/\beta}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \beta$
Variance	$Var(X) = \beta^2$

Normal Distribution, $X \sim N(\mu,\sigma)$
If X is any normal random variable where $X \sim N(\mu,\sigma)$, then the transformation $Z = \dfrac{X-\mu}{\sigma}$ yields a *standard* normal variable where $Z \sim N(0,1)$.

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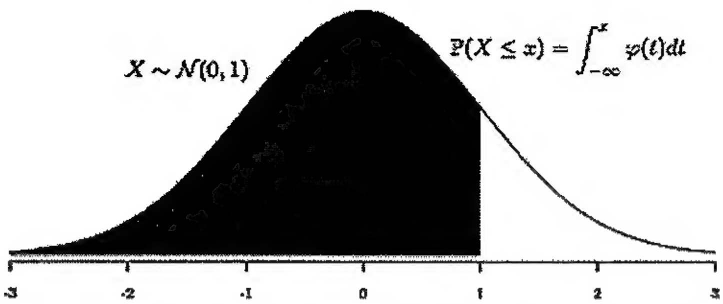
APPENDIX E:

Cumulative Standard Normal Distribution



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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